Black holes with regular horizons in Maxwell-scalar gravity.

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Abstract

A class of exact static spherically symmetric solutions of the Einstein-Maxwell gravity coupled to a massless scalar field has been obtained in harmonic coordinates of the Minkowski space-time. For each value of the coupling constant a, these solutions are characterized by a set of three parameters, the physical mass μ_0, the electric charge Q_0 and the scalar field parameter k. We have found that the solutions for both gravitational and electromagnetic fields are not only affected by the scalar field, but also the non-trivial coupling with matter constrains the scalar field itself. In particular, we have found that the constant k generically differs from $\pm 1/2$, falling into the interval $|k| \in [0, \frac{1}{2}\sqrt{1+a^2}\,]$. It takes these values only for black holes or in the case when a scalar field ϕ is totally decoupled from the matter. Our results differ from those previously obtained in that the presence of arbitrary coupling constant a gives an opportunity to rule out the non-physical horizons. In one of the special cases, the obtained solution corresponds to a charged dilatonic black hole with only one horizon μ_+ and hence for the Kaluza-Klein case. The most remarkable property of this result is that the metric, the scalar curvature, and both electromagnetic and scalar fields are all regular on this surface. Moreover, while studying the dilaton charge, we found that the inclusion of the scalar field in the theory result in a contraction of the horizon. The behavior of the scalar curvature was analyzed.

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1 INTRODUCTION.

Recently considerable interest has been shown in the physical processes occuring in the strong gravitational field regime. However, many modern theoretical models which include the general relativity as a standard gravity theory, are faced with the problem of the unavoidable appearence of space-time singularities. It is well known that the classical description, provided by general relativity, breaks down in a domain where the curvature is large, and, hence, a proper understanding of such regions requires new physics [1]. The tensor-scalar theories of gravity, where, the usual for general relativity tensor field, coexists together with one or several long-range scalar fields, are believed to be the most interesting extension of the theoretical foundation of modern gravitational theory. The superstring, many-dimensional Kaluza-Klein, and inflationary cosmology theories have revived the interest in so-called "dilaton fields", *i.e.* neutral scalar fields whose background values determine the strength of the coupling constants in the effective four-dimensional theory. However, although the scalar field naturally arises in theory, its existence leads to a violation of the strong equivalence principle and modification of large-scale gravitational phenomena [2], [3]. Moreover, the presence of the scalar field affects the

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equations of motion of the other matter fields as well. Thus, for example, the solutions to the Einstein-Maxwell-dilaton system were studied in [4]-[11], where it was shown that the scalar field generally destroys the horizons. This causes the singularities in a scalar curvature to appear on a finite radii. It is worth noting that the special attention has been paid to the charged dilatonic black hole solution presented in [8]. The analysis of this solution has shown that, in the case of $a = 0$, it reduces to Reisner-Nordström solution. However, for $a \neq 0$, this result represents qualitatively different physics. In particular, this solution has a regular outer event horizon but, for any non-zero value a, the inner horizon is singular. An interesting analogy of the behavior of the black holes and elementary particles has been demonstrated in [9]. Thus, by analyzing the perturbations around the extreme holes, the authors of this previous article have shown the existence of an energy gap in the excitation spectrum of the black hole, which corresponds to the potential barrier isolating them from the external world.

In order to resolve the dilatonic black-hole singularities, the higher-dimentional extension ($D \geq 4$) of the general relativity was considered in [5]. It was shown that a dilatonic black hole with a dilaton coupling constant $a = \sqrt{p/(p+2)}$ might be interpreted as a non-singular, non-dilatonic, black p-brane in $(4+p)$ dimentions. Moreover, when p is even, the p-brane resembles the extreme Reisner-Nordström solution in that there is still a curvature singularities inside the horizon. However, when p is odd, the solution inside the horizon is isomeric to that for the outside region, and it is completely non-singular. There the special interest is presenting the class of stationary, spherically symmetric black hole solutions in Kaluza-Klein theory with $a = \sqrt{3}$, which in four dimentions, was discussed and classified in [6]. The decay of magnetic fields in this theory and the possible mechanism of the pair creation of monopoles was analysed in [7]. However, even thought these solutions present interesting properties in higher dimentions, their geometry become singular at the classical level for $D = 4$.

The motivation for the present work was to find a stable dilatonic black hole solution, which would demonstrate non-singular properties for all possible interacting regimes in four dimentions. As we shall see later, the covariant generalization of the harmonic gauge presents the necessary opportunity. In this paper we will focus our attention on the simplest extension of the standard matter *i.e.* gravity coupled to interacting[2] electromagnetic and scalar fields. The density of the Lagrangian function L_M for the massless scalar and electromagnetic fields is suggested by the low-energy limit of the string theory and it has the usual form:

$$L_M = \frac{1}{16\pi}\sqrt{-g}\Big(- R + 2\nabla_n\phi\nabla^n\phi - e^{-2a\phi}F^2\Big), \tag{1}$$

where $F_{mn} = \nabla_m A_n - \nabla_n A_m$ is the tensor of the electromagnetic field[3]. The symmetries of this Lagrangian are the general covariance and the gauge symmetry. Besides this, the expression (1) is invariant under the global scale transformations, namely: $\phi'(x) = \phi(x) + \phi_c$ and $A'_m(x) = e^{a\phi_c}A_m(x)$. This freedom can be eliminated by specifying the value of the scalar field at infinity. The constant a in (1) is a dimensionless, arbitrary parameter. To study the dependence of the solutions on the strength of interaction between the scalar and electromagnetic fields, an arbitrary coupling constant a was introduced in [4], [8]. The arbitrariness of this constant makes it possible to have both weak ($a \ll 1$) and strong ($a \gg 1$) coupling regimes. For $a = 0$, Eq.(1) becomes the standard Einstein-Maxwell Lagrangian with an extra free massless scalar field. In the case $a = 1$, it corresponds to the contribution in total action from the low-energy limit of the

[2] The harmonic solution, presented in [11], was obtained in the special case $a = 0$ when the interaction between the matter fields is absent.

[3] The Plank units $\hbar = c = \gamma = 1$ are used throughout the paper and metric convention is accepted to be $(+ - --)$.

superstring theory, treated to the lowest order in world-sheet and string loop expansion. Since changing the sign of a is equivalent to changing the sign of ϕ, we will consider this theory just for $a \geq 0$.

In order to clearly state the new results obtained, the paper structured is as follows. In section 2 we will derive the basic system of equations for the gravitational, scalar and electromagnetic fields. The degree of arbitrariness caused by the covariant Fock - de Donder harmonic gauge will be discussed in section 3. The solutions for generalized radial coordinate and for the scalar field will be obtained in sections 4 and 5 respectievly. The general static spherically symmetric solution for interacting scalar and electromagnetic fields will be presented in section 6. Section 7 is devoted to analysis of the obtained solution in some special cases. The structure of the scalar curvature will be examined in section 8. The final results in parametric form will be presented in section 9, where we will summarize and suggest future directions for studying the behavior of the these solutions.

2 THE EQUATIONS OF MOTION.

By extremizing the Lagrangian function L_M (1) with respect to the metric g_{mn}, the electromagnetic potential A_m and the scalar field ϕ, one will obtain the corresponding field equations. Those for the gravitational field take the following form:

$$R_{mn} = 8\pi(T_{mn} - \frac{1}{2} g_{mn} T), \tag{2.1}$$

where the symmetric energy-momentum tensor of the matter fields T_{mn} is calculated to be:

$$T_{mn} = \frac{1}{4\pi}\Big(\nabla_m\phi\nabla_n\phi - \frac{1}{2}g_{mn}\nabla_k\phi\nabla^k\phi\Big) + \\ + \frac{e^{-2a\phi}}{8\pi}\Big(- 2F_m{}^k F_{nk} + \frac{1}{2}g_{mn}F^2\Big). \tag{2.2}$$

We will be looking for solutions of these equations which will admit the covariant generalization of the Fock - de Donder harmonical gauge [11], [12], namely:

$$\mathcal{D}_m\sqrt{-g}g^{mn} = 0, \tag{2.3}$$

where $\mathcal{D}_m$ is the covariant derivative with respect to Minkowskii metric γ_{mn}:

$$\gamma_{mn} = \mathrm{diag}\Big(1, -1, -r^2, -r^2\sin^2\theta\Big).$$

The equations of motion of the scalar and electromagnetic fields corresponding to L_M (1) may be written as follows:

$$\nabla_n\nabla^n\phi - \frac{a}{2}\, e^{-2a\phi}F^2 = 0, \tag{2.4}$$

$$\nabla_m\Big(e^{-2a\phi}F^{mn}\Big) = 0. \tag{2.5}$$

To avoid confusion, let us note that words "static spherically symmetrical" here imply that not only the electromagnetic field F_{mn}, but also the electromagnetic potential A_m, is spherically symmetrical and independent of time:

$$\phi(t, r, \theta, \varphi) = \phi(r), \qquad A(t, r, \theta, \varphi) = \Big(A_0(r), A_1(r), 0, 0\Big).$$

Imposing the same conditions on the scalar and gravitational fields, one may write the general form of the effective metric for the static spherical symmetric case as follows:

$$g_{mn} = \mathrm{diag}\Big(u(r), \ -v(r), \ -w(r), \ -w(r) \ \sin^2\theta \Big). \tag{2.6}$$

Then, finally, taking into account the definitions above, the system of the gravitational field equations (2.1) may be re-written as follows:

$$R_{00} = \frac{u''}{2v} + \frac{u'}{2v}(\frac{w'}{w} - \frac{v'}{2v} - \frac{u'}{2u}) = \frac{1}{v}(A_0')^2 e^{-2a\phi}, \tag{2.7a}$$

$$R_{11} = -\frac{u''}{2u} - \frac{w''}{w} + \frac{u'}{2u}(\frac{u'}{2u} + \frac{v'}{2v}) + \frac{w'}{w}(\frac{w'}{2w} + \frac{v'}{2v}) =$$

$$= 2(\phi')^2 - \frac{1}{u}(A_0')^2 e^{-2a\phi}, \tag{2.7b}$$

$$R_{22} = -\frac{w''}{2v} + \frac{w'}{2v}(\frac{v'}{2v} - \frac{u'}{2u}) + 1 = \frac{w}{uv}(A_0')^2 e^{-2a\phi}. \tag{2.7c}$$

The equation for the component R_{33} coincides with the one for R_{22}, and the other equations become exact equalities. Finally, the equations for the scalar and electromagnetic fields from Eqs.(2.4),(2.5) take the form:

$$\phi'' + \phi'(\frac{u'}{2u} - \frac{v'}{2v} + \frac{w'}{w}) = \frac{a}{u}(A_0')^2 e^{-2a\phi}, \tag{2.8}$$

$$(\frac{w}{\sqrt{uv}} A_0' e^{-2a\phi})' = 0. \tag{2.9}$$

The obtained system of equations (2.7)-(2.9) are all that one needs to find the general spherically symmetric solution to the Einstein-Maxwell-scalar system (1). In the next section we will examine the possibility of simplifying of this system of equations using the covariant gauge conditions (2.3).

3 THE PARAMETERIZATION OF THE SOLUTION.

The use of the covariant Fock - de Donder gauge conditions (2.3) significantly symplifyies the system of the field equations (2.7)-(2.9). To demonstrate this, let us make a linear combination of the first and third equations from the system (2.7) with the coefficients $1/u$ and $-1/w$ respectively. The right hand side of the obtained relation becomes zero as the matter fields fall out:

$$\frac{u''}{2u} + \frac{u'}{2u}(\frac{w'}{w} - \frac{v'}{2v} - \frac{u'}{2u}) + \frac{w''}{2w} + \frac{w'}{2w}(\frac{u'}{2u} - \frac{v'}{2v}) - \frac{v}{u} = 0. \tag{3.1}$$

From the gauge condition Eq.(2.3) one might get another, pure gravitational equation, namely:

$$(\sqrt{\frac{u}{v}} w)' = 2r\sqrt{uv}. \tag{3.2}$$

By defining new functions $\alpha(r)$ and $\beta(r)$ as: $\alpha = \sqrt{uv}$, $\beta = w\sqrt{u/v}$, one can get the following system of equations from the Eqs.(3.1) and (3.2):

$$\beta' = 2r\alpha, \qquad (\alpha'\beta^2/\alpha)' = 0. \tag{3.3}$$

The general solution of that system might be written in a parametric form. Indeed, let us present $\alpha(p)$ and $\beta(p)$ in a following way:

$$\alpha(p)=\frac{A}{r_p},\qquad \beta(p)=A(p^2-\mu^2)\cdot r_p,\qquad r_p=\frac{dr}{dp}, \tag{3.4}$$

where A, p and μ are constants (arbitrary for the moment). This substitution will enable us to eliminate the functions α and β from both equations (3.3) and, as a result, we will obtain two equations for the same function $r(p)$:

$$(p^2-\mu^2)\, r_{pp}+2p\, r_p-2r=0, \tag{3.5a}$$

$$(p^2-\mu^2)^2\, r_{pp}+\frac{B}{A^2}=0, \tag{3.5b}$$

where B is another arbitrary integrating constant. Equations (3.5) are easy to integrate; and the common solution for both of them may be presented as follows:

$$r(p)=q\Big[p+z_0\Big(p\ln\frac{p-\mu}{p+\mu}+2\mu\Big)\Big], \tag{3.6}$$

with arbitrary integrating constants q, z_0 and $B=4\mu^3 q z_0 A^2$. Then from (3.4), one may write the general solution for the system of Eqs.(3.3) in the parametric form as follows:

$$\alpha(p)=A\Big[1+z_0\Big(\ln\frac{p-\mu}{p+\mu}+\frac{2\mu p}{p^2-\mu^2}\Big)\Big]^{-1}, \tag{3.7a}$$

$$\beta(p)=A(p^2-\mu^2)\Big[1+z_0\Big(\ln\frac{p-\mu}{p+\mu}+\frac{2\mu p}{p^2-\mu^2}\Big)\Big], \tag{3.7b}$$

$$r(p)=q\Big[p+z_0\Big(p\,\ln\frac{p-\mu}{p+\mu}+2\mu\Big)\Big]. \tag{3.7c}$$

It was noted in [11] that the solution of the problem obtained for the partial case $z_0=0$ might be easily expanded to a general case with $z_0\neq 0$. Because of this, we will take $z_0=0$ from now on and will defer reconstructing a non-zero value of the constant z_0 to the final results. The constants A and q are multipliers which define the scale of measurements of the coordinate. Without losing generality, we may set these constants to be equal to unity.

The relations (3.7) enable one to express the variables u and v as follows:

$$u(r)=\frac{1}{v(r)}=\frac{r^2-\mu^2}{w(r)}. \tag{3.8}$$

It is worth noting that the first equation from those above is the usual Schwartzchild condition [8], however, the second equation, as we shall see later, will be responsible for qualitatievly different physics. By substituting this result into the system of equations (2.7)-(2.8), one obtains

$$[-\frac{w'}{w}(r^2-\mu^2)+2r]'=\frac{2Q^2}{w}e^{2a\phi}, \tag{3.9a}$$

$$-\frac{w''}{w}+\frac{1}{2}\Big(\frac{w'}{w}\Big)^2=2(\phi')^2, \tag{3.9b}$$

$$[\phi'(r^2-\mu^2)]' = \frac{aQ^2}{w}e^{2a\phi}. \tag{3.9c}$$

The electric charge Q is the integral of the Maxwell equations (2.9), which generalizes the Gauss' law for curved space-time in the following way:

$$E = \frac{Q}{w}e^{2a\phi}, \tag{3.10}$$

where $E = A_0'$ is the intensity of the electromagnetic field.

To find the solution for the function $w(r)$, let us define a new function $f(r)$ as follows:

$$w(r) = f(r)e^{2a\phi(r)}. \tag{3.11}$$

Using this expression, the system of the equations (3.9) may be rewritten as:

$$[-\frac{f'}{f}(r^2-\mu^2)+2r]'f = 2(1+a^2)Q^2, \tag{3.12a}$$

$$a\phi'' + a\phi'\frac{f'}{f} + (1+a^2)(\phi')^2 = -\frac{1}{2}\Big[\big(\frac{f'}{f}\big)' + \frac{1}{2}\Big(\frac{f'}{f}\Big)^2\Big], \tag{3.12b}$$

$$[\phi'(r^2-\mu^2)]'f = aQ^2. \tag{3.12c}$$

Our future strategy will be the following: first, we will solve the equation (3.12a) for the function f. Second, the obtained function f will be used in the equation (3.12b) (which is considered here as the equation for determining the scalar field ϕ). Solutions for the functions f and ϕ, obtained this way, should satisfy the equation of motion of the scalar field which is presented by the equation (3.12c).

4 THE SOLUTION FOR THE FUNCTION $f(r)$.

In order to find $f(r)$, we will introduce a new function $\nu(r)$ by the following relation:

$$f(r) = 2(1+a^2)Q^2\cdot(r^2-\mu^2)\nu^2(r), \tag{4.1}$$

Using this function, the equation (3.12a) can be presented in terms of $\nu(r)$ as:

$$[-(r^2-\mu^2)\frac{2\nu'}{\nu}]'(r^2-\mu^2)\nu^2 = 1. \tag{4.2}$$

After some algebra, one may obtain two solutions for this equation:

$$\nu_1(r) = \pm\frac{1}{4\sqrt{2}s\mu h}\Big[B\Big(\frac{r-\mu}{r+\mu}\Big)^{s+h} - \frac{1}{B}\Big(\frac{r+\mu}{r-\mu}\Big)^{s+h}\Big], \tag{4.3a}$$

$$\nu_0(r) = b \pm \frac{1}{2\sqrt{2}\mu}\ln\frac{r-\mu}{r+\mu}, \tag{4.3b}$$

The constants h, B, b and s are arbitrary for the moment and, in general, may have both real and imaginary values. It is easy to see that the result (4.3b) is simply the limiting case of the solution (4.3a) with parameter $h=0$, and therefore the expression for $\nu_1(r)$ from Eq.(4.3a) is

the general solution for the equation (4.2). Then the function $f_1(r)$ may be written from (4.1) and (4.3a) as follows:

$$f_1(r) = (1+a^2)\frac{Q^2}{16\mu^2 s^2 h^2}(r^2-\mu^2)\times$$

$$\times\Big[B\Big(\frac{r-\mu}{r+\mu}\Big)^{s+h} - \frac{1}{B}\Big(\frac{r+\mu}{r-\mu}\Big)^{s+h}\Big]^2. \tag{4.4}$$

Substituting this expression into Eq.(3.12a), one can see that function f_1 becomes the solution of this equation if the following condition is satisfied: $s^2h^2 = (s+h)^2 = k^2$, where k is some new arbitrary parameter. After this, the general solution for the function $f(r)$ may finally be presented by the expression:

$$f(r) = (1+a^2)\frac{Q^2}{16\mu^2 k^2}(r^2-\mu^2)\Big[B\Big(\frac{r-\mu}{r+\mu}\Big)^{k} - \frac{1}{B}\Big(\frac{r+\mu}{r-\mu}\Big)^{k}\Big]^2. \tag{4.5a}$$

In the limit $k=0$, this result will take the form:

$$f_0(r) = 2(1+a^2)Q^2(r^2-\mu^2)\left(b \pm \frac{1}{2\sqrt{2}\mu}\ln\frac{r-\mu}{r+\mu}\right)^2. \tag{4.5b}$$

5 THE SOLUTION FOR THE SCALAR FIELD $\phi(r)$.

To find the solution for the function ϕ from the equation (3.12b) we will use the following substitution:

$$\phi'(r) = \frac{\xi(r)}{r^2-\mu^2}, \tag{5.1}$$

where $\xi(r)$ is a new function to be determined. Then, with the help of the expressions (4.3a) and (4.5a), the equation Eq.(3.12b) becomes:

$$a(r^2-\mu^2)(\xi' + 2\xi\frac{\nu_1'}{\nu_1}) + (1+a^2)\xi^2 = \mu^2(1-4k^2). \tag{5.2}$$

Let us define a new radial coordinate z as follows:

$$z = \frac{B^2\rho^2-1}{B^2\rho^2+1}, \qquad \rho = \left(\frac{r-\mu}{r+\mu}\right)^k. \tag{5.3}$$

Then the equation (5.2) may be re-written as:

$$\xi_z(1-z^2) + \frac{2}{z}\xi + \frac{1+a^2}{2\mu k a}\xi^2 = \frac{\mu}{2ka}(1-4k^2). \tag{5.4}$$

The general solution of this differential equation has the following form:

$$\xi(\rho) = \frac{2\mu k a}{1+a^2}\left(\frac{\delta}{k}\,\frac{C_0^2(B\rho)^{2\delta/k}+1}{C_0^2(B\rho)^{2\delta/k}-1} - \frac{(B\rho)^2+1}{(B\rho)^2-1}\right), \tag{5.5}$$

where C_0 is arbitrary integrating constant and δ is given by the expression:

$$\delta = \pm\frac{1}{2a}\sqrt{1+a^2-4k^2}. \tag{5.6}$$

This finally gives the following general solution for the function $\phi(r)$:

$$\phi(r)=\phi_0-\frac{a}{1+a^2}\ln\Big[B\Big(\frac{r-\mu}{r+\mu}\Big)^k-\frac{1}{B}\Big(\frac{r+\mu}{r-\mu}\Big)^k\Big]+$$

$$+\frac{a}{1+a^2}\ln\Big[C_0B^{\delta/k}\Big(\frac{r-\mu}{r+\mu}\Big)^\delta-\frac{1}{C_0B^{\delta/k}}\Big(\frac{r+\mu}{r-\mu}\Big)^\delta\Big], \qquad (5.7)$$

where ϕ_0 is an arbitrary integrating constant.

6 THE GENERAL SOLUTION.

Now we are in a position to obtain the general solution for the function $w(r)$. By substituting the expression (4.7a) into Eq.(3.11) and expressing $e^{2a\phi(r)}$ with the help of the relation (5.7) we may write the result for $w(r)$ as follows:

$$w(r)=(1+a^2)\frac{Q^2}{16\mu^2k^2}e^{2a\phi_0}(r^2-\mu^2)\times$$

$$\times\Big[B\Big(\frac{r-\mu}{r+\mu}\Big)^k-\frac{1}{B}\Big(\frac{r+\mu}{r-\mu}\Big)^k\Big]^{\frac{2}{1+a^2}}\times$$

$$\times\Big[C_0B^{\delta/k}\Big(\frac{r-\mu}{r+\mu}\Big)^\delta-\frac{1}{C_0B^{\delta/k}}\Big(\frac{r+\mu}{r-\mu}\Big)^\delta\Big]^{\frac{2a^2}{1+a^2}}, \qquad (6.1)$$

where the constants μ, Q, k, B, C_0 and ϕ_0 are arbitrary for the moment. In order to limit the number of arbitrary constants in this solution, we will impose two asymptotical conditions on functions $\phi(r)$ and $w(r)$, namely:

$$\lim_{r\to\infty}\phi(r)=\phi_0, \qquad \lim_{r\to\infty}w(r)=r^2. \qquad (6.2)$$

By applying the first of the conditions (6.2) and with the help of the relation (5.7), we have:

$$C_0B^{\delta/k}-\frac{1}{C_0B^{\delta/k}}=B-\frac{1}{B}. \qquad (6.3)$$

Making of use the second condition from Eq.(6.2) and taking into account the result above, we will obtain another constraint:

$$\Big(B-\frac{1}{B}\Big)^{-2} = (1+a^2)\frac{Q^2e^{2a\phi_0}}{16\mu^2k^2}. \qquad (6.4)$$

These results enable us to write the general solution for the function $w(r)$. Thus from the expression (6.1) with the help of the relations (6.3) and (6.4) we will obtain this function in the following form:

$$w(r)=(r^2-\mu^2)\Big[\frac{B^2}{B^2-1}\Big(\frac{r-\mu}{r+\mu}\Big)^k-\frac{1}{B^2-1}(\frac{r+\mu}{r-\mu})^k\Big]^{\frac{2}{1+a^2}}\times$$

$$\times\Big[\frac{C_0^2B^{2\delta/k}}{C_0^2B^{2\delta/k}-1}\Big(\frac{r-\mu}{r+\mu}\Big)^\delta-\frac{1}{C_0^2B^{2\delta/k}-1}(\frac{r+\mu}{r-\mu})^\delta\Big]^{\frac{2a^2}{1+a^2}}, \qquad (6.5)$$

where the constant B may be defined by Eq.(6.4) as below:

$$B = \pm\frac{1}{A}\Big(1 \pm \sqrt{1+A^2}\Big), \qquad A^2 = (1+a^2)\frac{Q^2 e^{2a\phi_0}}{4\mu^2 k^2}. \tag{6.6}$$

Making of use the result for the constants B and C_0 given by Eq. (6.3), one might present the final solution for the function $\phi(r)$ as well. Thus, from the expression (5.7) we will have

$$\phi(r) = \phi_0 - \frac{a}{1+a^2}\ln\Big[\frac{B^2}{B^2-1}\Big(\frac{r-\mu}{r+\mu}\Big)^k - \frac{1}{B^2-1}\Big(\frac{r+\mu}{r-\mu}\Big)^k\Big] +$$

$$+\frac{a}{1+a^2}\ln\Big[\frac{C_0^2 B^{2\delta/k}}{C_0^2 B^{2\delta/k}-1}\Big(\frac{r-\mu}{r+\mu}\Big)^\delta - \frac{1}{C_0^2 B^{2\delta/k}-1}\Big(\frac{r+\mu}{r-\mu}\Big)^\delta\Big]. \tag{6.7}$$

We have obtained a general solution for the system of equations Eqs.(3.9). For arbitrary values of the coupling constant a, this solution is labeled by five arbitrary parameters μ, Q, k, ϕ_0 and C_0. This number of arbitrary parameters contradicts the "no-hair" theorem, which states that no parameters other than mass, electric charge, and angular momentum may be associated with a black hole. Moreover, the parameters, entering the solution, should correspond to three conserved Noether currents for the fields involved, which are easy to obtain from the Lagrangian (1). Having this in mind, we will analyse the obtained general solution Eqs.(6.5)-(6.7) in the next section in order to obtain a physically reasonable condition to limit the arbitrariness associated with these parameters.

7 THE ANALYSIS OF THE SPECIAL CASES.

In this section we analyse the various special cases of the general solution by setting some of the parameters equal to zero while the others remain unchanged.

(i). $a = 0$. In this case the result obtained represents the solution for the Einstein-Maxwell system with an extra free scalar field. The dependence on the constant C_0 in Eqs.(6.5)-(6.7) drops out and the solution may be labeled by the set of three parameters (μ, k and Q_0):

$$\phi(r)|_{a=0} = \phi_0 \pm \frac{1}{2}\sqrt{1-4k^2}\ln\frac{r-\mu}{r+\mu}, \tag{7.1a}$$

$$w(r)|_{a=0} = \frac{1}{4}(r^2-\mu^2)\times$$

$$\times\Big[\Big(1 \pm \sqrt{1+A_0^2}\Big)\Big(\frac{r-\mu}{r+\mu}\Big)^k + \Big(1 \mp \sqrt{1+A_0^2}\Big)\Big(\frac{r+\mu}{r-\mu}\Big)^k\Big]^2, \tag{7.1b}$$

where the constant A_0 is given by the expression for A (6.6) with $a = 0$ This result corresponds to that obtained in [11]. One may notice that the scalar field is real for $|k| \le 1/2$, but becomes complex when $|k| > 1/2$. Using the relations for the metric functions (3.8) and (2.6), we will obtain the actual form of the interval ds^2. Thus, for example, for $k = \pm 1/2$, one may notice that the scalar field vanishes (or it becomes constant $\phi = \phi_0$) and the solution takes the form:

$$ds^2 = \frac{r^2-\mu^2}{\Big(r+\sqrt{\mu^2+Q^2}\Big)^2}dt^2 -$$

$$-\frac{\left(r+\sqrt{\mu+Q^2}\right)^2}{r^2-\mu^2}dr^2-\left(r+\sqrt{\mu^2+Q^2}\right)^2 d\Omega, \qquad (7.2a)$$

where $d\Omega = d\theta^2 + \sin^2\theta d\varphi^2$. The parameter $\mu > 0$ is related to the physical mass μ_0 as:

$$\mu = \sqrt{\mu_0^2 - Q^2}. \qquad (7.2b)$$

By re-writing the expression (7.2a) in terms of the physical mass μ_0, we esstablish the correspondence of this result to the solution of the Reisner-Nordström type, obtained in harmonic coordinates of the Minkowski space-time, which may be presented as follows:

$$ds^2 = \Big(\frac{r-\mu_0}{r+\mu_0} + \frac{Q^2}{(r+\mu_0)^2}\Big)dt^2 -$$

$$-\Big(\frac{r-\mu_0}{r+\mu_0} + \frac{Q^2}{(r+\mu_0)^2}\Big)^{-1}dr^2 - (r+\mu_0)^2 d\Omega. \qquad (7.2c)$$

This result, unlike the usual Reisner-Nordström solution, has one horizon only, which is given by the expression (7.2b). This seems to be quite reasonable. Indeed, from the beginning we have been looking for a solution outside the source where the energy-momentum tensor of matter distribution equals zero. As a result, one may show that the physical radius of the horizon $r_0(\mu)$ is positive and always remains outside the body's mass shell $r_0(\mu) \geq \mu_0$, which is unlike the conventional Reisner-Nordström solution, where the "inner" horizon is always inside the mass surface $r_- \leq \mu_0$ and equal to it in the extreme case only.

(ii). An interesting case is arises in the strong interaction regime when $a \gg 1$. Examining the expressions (6.5)-(6.6) in the extreme regime of $a \to \infty$ and $k = \pm 1/2$ one obtains $\phi(r) = \phi_0$ and the following expression for $w(r)$:

$$w(r) = (r^2-\mu^2)\Big[\frac{C_0^2}{C_0^2-1}\Big(\frac{r-\mu}{r+\mu}\Big)^{\pm 1/2} - \frac{1}{C_0^2-1}\Big(\frac{r+\mu}{r-\mu}\Big)^{\pm 1/2}\Big]^2. \qquad (7.3a)$$

This solution corresponds to that of the Reisner-Nordström type with the "induced charge" J generated by the constant C_0. Taking, for example, the minus sign in the powers of expression in (7.3a), one obtains the following expression:

$$ds^2 = \Big(\frac{r-\hat\mu}{r+\hat\mu} + \frac{J^2}{(r+\hat\mu)^2}\Big)dt^2 -$$

$$-\Big(\frac{r-\hat\mu}{r+\hat\mu} + \frac{J^2}{(r+\hat\mu)^2}\Big)^{-1}dr^2 - (r+\hat\mu)^2 d\Omega, \qquad (7.3b)$$

where parameters μ and C_0 are connected to physical mass $\hat\mu$ and "charge" J as follows:

$$\mu = \hat\mu\frac{C_0^2-1}{C_0^2+1} \qquad J = \frac{2\hat\mu C_0}{C_0^2+1}.$$

This result is quite surprising. Indeed, taking the limit $a \to \infty$ and the condition $k = \pm 1/2$ is equivalent to cutting off both the electromagnetic and the scalar terms in the Lagrangian density L_M (1). Then, because of no matter fields are present, this solution should be one for a pure static spherically symmetric gravity. Instead, as a result, one obtains the solution (7.3b) of the Reisner-Nordström type with the effective metric similar to that in (7.2c). Since the scalar

field is responsible for appearance of the constant C_0, then the "induceed charge" J is caused by the scalar field, which is absent! In order to resolve this apparent paradox, we must require that $C_0 = 0$. This condition simply corresponds to the renormalization of the constant ϕ_0 in Eq.(6.1). Then the expression (7.3b) becomes the Fock solution for static spherically symmetric black hole in harmonic coordinates of the Minkowskii space-time:

$$ds^2 = \Big(\frac{r-\mu}{r+\mu}\Big)dt^2 - \Big(\frac{r+\mu}{r-\mu}\Big)dr^2 - (r+\mu)^2 d\Omega. \tag{7.4}$$

This solution has one horizon at $\mu \Rightarrow \mu_+ = \mu_0$. Note, the physical radius of the horizon is always positive and equals $r_0(\mu) = 2\mu_0$.

(iii). $Q = 0$. When the electric charge vanishes, the solution reduces to that of pure scalar gravity with interval ds^2 written as:

$$ds^2 = \Big(\frac{r-\mu}{r+\mu}\Big)^q dt^2 - \Big(\frac{r+\mu}{r-\mu}\Big)^q dr^2 - (r^2-\mu^2)\Big(\frac{r+\mu}{r-\mu}\Big)^q d\Omega, \tag{7.5a}$$

where the constant q given by the relation

$$q = 2\,\frac{k+a^2\delta}{1+a^2} = \frac{1}{1+a^2}\Big(2k \pm a\sqrt{1+a^2-4k^2}\,\Big). \tag{7.5b}$$

The scalar field $\phi(r)$ for this case is represented by the following expression:

$$\phi(r) = \phi_0 + \frac{1}{2(1+a^2)}\Big(2ak \pm \sqrt{1+a^2-4k^2}\Big)\ln\Big(\frac{r-\mu}{r+\mu}\Big). \tag{7.5c}$$

The parameter $\mu \geq 0$ defines the location of one horizon (μ_+) which is, in the case (7.5), related to the physical mass μ_0 as $\mu_+ = \mu_0/q$ and, for any $q \neq 1$, this horizon is singular[4].

Note that taking $Q = 0$ is equivalent to dropping the electromagnetic term from the Lagrangian density L_M (1). However, one might find quite unexpectedly that, even after taking $Q \to 0$, our results still depend on the arbitrary parameter a which characterizes the intensity of the interaction between the matter fields. This contradiction might be resolved by both taking the parameter k to be $k = \pm 1/2$ and by choosing the signs in (7.5) in such a way that these expressions will not depend on a. This suggests that it is not only the scalar field that affects the solutions for the gravitational and electromagnetic fields, but also the interaction between the matter fields that puts constraints on the scalar field itself. The usual Fock solution (7.4) might be obtained from (7.5) by setting $q = 1$ and choosing the same signs for both terms in (7.5b).

(iv). One might expect that all the expressions for the general solution should omit the homogeneous non-trivial limit in the case where constant a becomes imaginary: $a \to \pm i$. In that limit one will obtain the following assymptotically flat (with $\phi_0 = 0$) result:

$$\phi(r)\big|_{a\to\pm i} = \mp i\Big[\frac{1-4k^2}{8k}\,\ln\frac{r+\mu}{r-\mu} + \\ + \frac{Q^2}{16\mu^2k^2}\Big(1 - \Big(\frac{r-\mu}{r+\mu}\Big)^{2k}\Big)\Big], \tag{7.6a}$$

$$w(r)\big|_{a\to\pm i} = (r^2-\mu^2)\Big(\frac{r+\mu}{r-\mu}\Big)^{\frac{1+4k^2}{4k}}\exp\Big[\frac{Q^2}{8\mu^2k^2}\Big(1 - \Big(\frac{r-\mu}{r+\mu}\Big)^{2k}\Big)\Big]. \tag{7.6b}$$

[4] The condition $q = 1$ may be written equivalently as $(2k \pm 1)^2(1+a^2) = 0$.

It is easy to see that these expressions are, in general, singular. However, choosing the parameter $k = \pm 1/2$, one might avoid this singularity. Thus, for $k = 1/2$ the expressions (7.6) become:

$$\phi(r) = \mp i \, \frac{Q^2}{4\mu^2}\Big(1 - \frac{r-\mu}{r+\mu}\Big), \tag{7.7a}$$

$$w(r) = (r+\mu)^2 \exp\Big[\frac{Q^2}{2\mu^2}\Big(1 - \frac{r-\mu}{r+\mu}\Big)\Big], \tag{7.7b}$$

$$u(r) = \frac{1}{v(r)} = \Big(\frac{r-\mu}{r+\mu}\Big) \exp\Big[\frac{Q^2}{2\mu^2}\Big(\frac{r-\mu}{r+\mu} - 1\Big)\Big]. \tag{7.7c}$$

This is an interesting generalization of the Fock solution (7.4) in the presence of the electromagnetic and pure complex scalar fields. Solution (7.7) is labeled by the means of two parameters μ and Q. This solution has regular event horizon r_+, which is related to the physical mass μ_0 and the charge Q of the black hole as follows:

$$r_+ \quad \Rightarrow \quad \mu_+ = \frac{1}{2}\Big(\mu_0 + \sqrt{\mu_0^2 - 2Q^2}\,\Big). \tag{7.8}$$

The expression (7.8) limits the possible value of the physical mass to be $\mu_0 \geq \sqrt{2}|Q|$. It is easy to show that the scalar curvature corresponding to the solution (7.7) is also regular on surface (7.8).

The presence of i in the expression for ϕ in (7.7a) might be interpreted as changing the sign in front of the scalar field's kinetic term [13] in the Lagrangian density L_M (1) to be:

$$L_M = -\frac{1}{16\pi}\sqrt{-g}\Big(R + 2g^{mn}\nabla_m\varphi\nabla_n\varphi + e^{-2\varphi}g^{mn}g^{kl}F_{mk}F_{nl}\Big), \tag{7.9}$$

where we denote $\varphi = -i\phi$. Unfortunately, the negative kinetic term $-g^{mn}\nabla_m\varphi\nabla_n\varphi$ in (7.9) generally leads to a theory without stable ground state and, moreover, it allows infinitely many negative energy states when the system is quantized [3], [11].

(v). In the case of $k = 0$ the general solution of Eqs.(6.5)-(6.7) becomes:

$$\phi(r)|_{k=0} = \phi_0 - \frac{1}{2}\frac{1}{\sqrt{1+a^2}} \ln\Big(\frac{r-\mu}{r+\mu}\Big) -$$

$$-\frac{a}{1+a^2}\ln\Big[1 - \sqrt{1+a^2}\frac{Q_0}{2\mu}\ln\Big(\frac{r-\mu}{r+\mu}\Big)\Big], \tag{7.10a}$$

$$w(r)|_{k=0} = (r^2 - \mu^2)\times$$

$$\times\Big[1 - \sqrt{1+a^2}\frac{Q_0}{2\mu}\ln\Big(\frac{r-\mu}{r+\mu}\Big)\Big]^{\frac{2}{1+a^2}} \Big(\frac{r+\mu}{r-\mu}\Big)^{\frac{a}{\sqrt{1+a^2}}}. \tag{7.10b}$$

With $Q_0 = 0$ and an arbitrary a, this result represents the usual solution for the scalar field given by (7.5) with $k = 0$. For an arbitrary value of both parameters a and Q_0, the expressions (7.10) represent the solution with the singularity at $r = \mu$. The parameter μ is related to physical mass μ_0 and electric charge $Q_0 = Qe^{a\phi_0}$ as follows:

$$a\mu = \mu_0\sqrt{1+a^2} - Q_0. \tag{7.10c}$$

Taking into account that parameters a and μ are both non-negative $a, \mu > 0$, one may conclude that this last relation saturates the bound $\mu_0\sqrt{1+a^2} \geq Q_0$. If we further set the parameter $\mu = 0$ (extreme case), the result (7.10) will take the form:

$$\phi(r)\Big|_{\substack{\mu=0\\k=0}} = \phi_0 - \frac{a}{1+a^2}\ln\Big[1 + (1+a^2)\frac{\mu_0}{r}\Big], \tag{7.11a}$$

$$w(r)\Big|_{\substack{\mu=0\\k=0}} = r^2\Big[1 + (1+a^2)\frac{\mu_0}{r}\Big]^{\frac{2}{1+a^2}}. \tag{7.11b}$$

It easy to see from (7.10c), that the physical mass μ_0 is, in this case, generated by the electric charge only. For any $a \neq 0$, this expression is singular at $r = 0$, however, in the case $a = 0$ result (7.11) corresponds to extreme case of the Reisner-Nordström solution (7.2c).

(vi). And finally, in the case $k = \pm 1/2$, the general solution becomes a charged dilatonic black hole solution in harmonic coordinates. To show this, let us take, for example, $k = 1/2$ and a negative sign in front of the $\sqrt{1+A^2}$ in (6.6). Then one will obtain the following result[5]:

$$\phi(r) = \phi_0 + \frac{a}{1+a^2}\ln\left(\frac{r+\mu}{r+\sqrt{\mu^2+Q_\star^2}}\right), \tag{7.12a}$$

$$w(r) = \left(r + \sqrt{\mu^2+Q_\star^2}\right)^2\left(\frac{r+\mu}{r+\sqrt{\mu^2+Q_\star^2}}\right)^{\frac{2a^2}{1+a^2}}, \tag{7.12b}$$

$$u(r) = \frac{1}{v(r)} = \left(\frac{r-\mu}{r+\sqrt{\mu^2+Q_\star^2}}\right)\left(\frac{r+\mu}{r+\sqrt{\mu^2+Q_\star^2}}\right)^{\frac{1-a^2}{1+a^2}}, \tag{7.12c}$$

where the parameter $\mu \geq 0$ and the constant $Q_\star^2$ is defined by Eq. (6.6) as:

$$Q_\star^2 = (1+a^2)Q^2e^{2a\phi_0}. \tag{7.13}$$

This solution, unlike the result discussed in [8], has only one horizon $\mu_+ \geq 0$, which is given as follows:

$$\mu_+ = \frac{1}{1-a^2}\Big(\sqrt{\mu_0^2 - (1-a^2)Q_0^2} - \mu_0 a^2\Big). \tag{7.14a}$$

In the case $a = 0$ this result corresponds to that of Eq.(7.2b) and for $a = 1$ one will have:

$$\mu_+\Big|_{a=1} = \mu_0 - \frac{Q_0^2}{2\mu_0}. \tag{7.14b}$$

The expression (7.14a) saturates the bound for the charge $Q_0 = Qe^{a\phi_0}$ of the hole as presented below:

$$\mu_0^2(1+a^2) \geq Q_0^2 \tag{7.14c}$$

Furthermore, one may show that horizon μ_+ is the regular function of the coupling parameter a, so that the function $\mu_+(a)$ remains non-negativite $\mu_+ \geq 0$ and finite $\mu_+ \leq \mu_{+max} < \infty$ for all values of a: $a \in [0, \infty[$. The physical radius of the horizon $r_0(\mu)$ is defined by the expression $r_0^2(\mu) = w(r)_{r=\mu}$, which in terms of the physical mass μ_0 and charge Q_0 gives the following result:

$$r_0^2(\mu) = \left(\mu_0 + \sqrt{\mu_0^2 - (1-a^2)Q_0^2}\right)^{\frac{2}{1+a^2}} \times$$

[5]Result for the value $k = -1/2$ might be obtained by changing $\mu \to -\mu$ in the expressions (12) and will correspond to a solution with a negative physical mass.

$$\times\left(\frac{2}{1-a^2}\Big[\sqrt{\mu_0^2-(1-a^2)Q_0^2}-2\mu_0 a^2\Big]\right)^{\frac{2a^2}{1+a^2}}. \tag{7.15a}$$

Thus, in the case $a=0$, the radius r_0 is given as follows:

$$r_0(\mu)\Big|_{a=0}=\mu_0+\sqrt{\mu_0^2-Q^2}. \tag{7.15b}$$

This expression always remains finite and reaches its minimum value μ_0 for the extreme case, when $\mu_0=|Q|$. When $a=1$, the expression (7.15a) behaves as:

$$r_0(\mu)\Big|_{a=1}=2\mu_0\left(1-\frac{Q_0^2}{2\mu_0^2}\right)^{1/2}. \tag{7.15c}$$

Note, that for any $a>0$ the radius $r_0(\mu)$ vanishes for the extreme hole, and the geometry becomes singular.

Concluding this section we would like to emphasise that in order for the general solution (Eqs.(6.5) and (6.7)) to correspond to a black hole, it may contain only three parameters, namely: μ, a and $Qe^{a\phi_0}$. This conclusion is in accord with the "no-hair" theorem and, in the static spherically symmetric case, instead of the angular momentum, the coupling constant a may be added to the set of the possible parameters. This result may be interpreted as if the conserved dilaton charge D becomes an additional parameter of the solution. The dilaton charge is defined by the statement that at infinity $\phi(r)\to\phi_0+D/r+\mathcal{O}(r^{-2})$, which for the case (7.12) gives the following result:

$$D=\frac{a}{1-a^2}\Big(\sqrt{\mu_0^2-(1-a^2)Q_0^2}-\mu_0\Big). \tag{7.16}$$

One can see that this expression is always negative. It vanishes in the case $a=0$, for $a=1$ it becomes $D=-Q_0^2/(2\mu_0)$, and it approaches the assymphotic value, $D_\infty=-|Q_0|$, when $a\to\infty$. Note, that D is not an independent parameter, however, if one decides to account for it as being independent, one will obtain an interesting modification of the expression (7.14a) for the horizon of the solution:

$$\mu_+=\mu_0-\frac{|D|}{a}. \tag{7.17}$$

This result means that inclusion of the scalar field in the theory is leading to a contraction of the horizon. The usual condition $\mu\geq 0$ saturates the boundary for the reduced dilaton charge $\hat{D}_a=|D|/a$ as $\mu_0\geq\hat{D}_a$. The horizon vanishes for the extreme case, when $\hat{D}_a=\mu_0$, leading to a naked singularity.

It is easy to verify that in the limit $Q\to 0$, the expressions (7.12) correspond to the Fock solution (7.4) independent of the value for the constant a. Indeed, taking $Q=0$ is equivalent to extracting the electromagnetic term from the action (1). Moreover, by choosing the parameter k to be $k=\pm 1/2$, we will also eliminate the term corresponding to a free scalar field. Then, the solution in this limit should describe a static spherically symmetric distribution of matter. This analysis suggests that the result (7.12) corresponds to a charged dilatonic black hole solution in harmonic coordinates. In the next section we will show that not only the metric, the electromagnetic field and the scalar field of the solution (7.12) are regular on the surface (7.14a), but, in addition to this, the scalar curvature $R(r)$ is also remains finite.

8 THE SCALAR CURVATURE.

It is well known that the simplest way to study the behavior of the scalar curvature R is to use the gravitational field equations. Indeed, as long as the electromagnetic part of the energy-momentum tensor Eq.(2.2) is traceless, the only contribution to the curvature R comes from the scalar field ϕ. Thus, by taking the trace of the Hilbert-Einstein equations (2.1), one can present the scalar curvature R as:

$$R(r) = -8\pi T = 2g^{mn}\nabla_m\phi\nabla_n\phi = -\frac{2\phi'^2(r)}{w(r)}(r^2-\mu^2). \tag{8.1}$$

Substituting the results for $\phi(r)$ and $w(r)$ from the general solution presented by the Eqs. (6.5) and (6.7) in the expression above, we will obtain the expression for the corresponding scalar curvature R as follows:

$$R(r) = -\frac{8a^2}{(1+a^2)^2}\frac{\mu^2}{(r^2-\mu^2)^2}\Big(\frac{r-\mu}{r+\mu}\Big)^q \times$$

$$\times\Big[\frac{1}{B_-\left(\frac{r-\mu}{r+\mu}\right)^{2k}+B_+}\Big]^{\frac{2}{1+a^2}}\Big[(\delta-k)-\frac{2kB_-\left(\frac{r-\mu}{r+\mu}\right)^{2k}}{B_-\left(\frac{r-\mu}{r+\mu}\right)^{2k}+B_+}\Big]^2, \tag{8.2}$$

where the constants δ and q are defined by (5.6) and (7.5b) respectively. The parameters $B_\pm$ and A are given as:

$$B_\pm = \frac{1}{2}\Big(1\pm\sqrt{1+A^2}\Big), \qquad A^2 = (1+a^2)\frac{Q^2e^{2a\phi_0}}{4\mu^2m^2}. \tag{8.3}$$

The expression (8.2) shows that the scalar curvature R is generically divergent on the surface: $r=\mu$. However, in two special cases one might get finite results, namely: (i) when $q=2$ [6], and (ii) when $k=\delta=\pm 1/2$. As we have seen, the most interesting properties are demonstrated by the solution (7.12). This solution has one horizon μ_+ where the metric and both electromagnetic and scalar fields are regular. Moreover, it is easy to verify that the corresponding scalar curvature $R(r)$ is also regular and has the following form:

$$R(r) = -\frac{2a^2}{(1+a^2)^2}\Big(\frac{r-\mu}{r+\mu}\Big)\times$$

$$\times\frac{(\sqrt{\mu^2+Q_\star^2}-\mu)^2}{(r+\sqrt{\mu^2+Q_\star^2})^4}\left(\frac{r+\sqrt{\mu^2+Q_\star^2}}{r+\mu}\right)^{\frac{2a^2}{1+a^2}}, \tag{8.4}$$

where constant $Q_\star$ is defined by the expressions (7.13). As it might be expected, $R(r)$ tends to be zero at the limits $Q\to 0$ and $a\to 0$. Note that the scalar curvature (8.4) becomes zero on the horizon $r=\mu=\mu_+$ and then changes sign for $r<\mu_+$. This suggests that our solution describes the exterior of the charged black hole only, *i.e.* the region outside the horizon for which $r>\mu_+$. To analyze the behavior of the solution (7.12) for distances $r\le\mu_+$, one should choose the model of matter distribution inside the star, obtain the solution for corresponding field equations in the interior region of the star, and then match both solutions on the surface. This research is in progress and will be reported elsewhere.

[6] The condition $q=2$ is equvivalent to the following equation: $(k-1)^2+\frac{3}{4}a^2=0$.

9 DISCUSSION.

Concluding this paper we would like to present the final form of the general static spherically symmetric harmonic solution of the Einstein-Maxwell gravity coupled to the massless scalar field. By reconstructing the constant z_0 from (3.7c) we will write this solution in the following parametric form:

$$ds^2 = \frac{p^2-\mu^2}{w(p)}dt^2 - \frac{w(p)}{p^2-\mu^2}r_p^2 dp^2 - w(p)d\Omega, \tag{9.1}$$

$$E(p) = \frac{Qr_p}{w(p)}e^{2a\phi(p)}, \tag{9.2}$$

where

$$r_p = \frac{dr}{dp} = 1 + z_0\Big(\ln\frac{p-\mu}{p+\mu} + \frac{2\mu p}{p^2-\mu^2}\Big).$$

The functions ϕ and w from (6.5) and (6.7) will be finally:

$$\phi(p) = \phi_0 - \frac{a}{1+a^2}\ln\Big[\Big(\frac{p+\mu}{p-\mu}\Big)^{(k-\delta)}\Big(B_-\Big(\frac{p-\mu}{p+\mu}\Big)^{2k} + B_+\Big)\Big], \tag{9.3a}$$

$$w(p) = (p^2-\mu^2)\Big(\frac{p+\mu}{p-\mu}\Big)^q\Big[B_-\Big(\frac{p-\mu}{p+\mu}\Big)^{2k} + B_+\Big]^{\frac{2}{1+a^2}}, \qquad 9.3b)$$

with the constants $B_\pm$ defined by the expressions (8.3).

Thus, we have presented the class of the solutions (9.3) which describes the exterior region of the black holes and the naked singularities and, as any solution, obtained with the harmonical gauge condition (2.3), this result has only one horizon μ_+, which is related to physical mass μ_0 and the electric charge $Q_0 = Qe^{a\phi_0}$ as follows:

$$\mu_+ = \frac{1}{4k^2-a^2}\Big[\sqrt{4\mu_0^2k^2-(4k^2-a^2)Q_0^2} - \mu_0 a\sqrt{1+a^2-4k^2}\Big]. \tag{9.4a}$$

This expression, independent of the value of the parameter k, saturates the usual bound $\mu_0^2(1+a^2) \geq Q_0^2$. Moreover, the condition $1+a^2-4k^2 \geq 0$ suggests that, for an arbitrary values of the coupling constant a, the physically interesting solutions do not exist if the parameter k belongs to $|k| > \frac{1}{2}\sqrt{1+a^2}$. Thus, for each value of the coupling constant a, this solution is characterized by a set of three parameters, the physical mass μ_0, the electric charge Q_0 and the scalar field parameter k. Note that for a spherical body whose radius is larger than μ, the constant k generically differs from $\pm 1/2$ and falls into the interval $|k| \in \Big[0, \frac{1}{2}\sqrt{1+a^2}\Big]$. As we saw, it takes these values only for black holes or in the case when a scalar field ϕ is totally decoupled from the matter (besides photons). The dilaton charge D corresponding to the solution (9.3) is given as

$$D = \frac{1}{4k^2-a^2}\Big[\sqrt{1+a^2-4k^2}\sqrt{4\mu_0^2k^2-(4k^2-a^2)Q_0^2} - \mu_0 a\Big]. \tag{9.4b}$$

Taking D as an independent parameter of the solution (instead of Q), one can present the horizon (9.4a) as follows:

$$\mu_+ = \frac{\mu_0 a + D}{\sqrt{1+a^2-4k^2}}. \tag{9.4c}$$

We have shown that because this solution was obtained in the vacuum outside the matter distribution, the result (9.3) describes the region which lies outside the surface (9.4a) only, *i.e.* for $r > \mu_+$. Although including a scalar field in the theory drastically affects the space-time

geometry and, in general, destroys the horizons, the presence of arbitrary coupling constant a gives an opportunity to explore the behavior of the results obtained in a different interaction regimes. Thus, we have found that the solutions for both gravitational and electromagnetic fields are not only affected by the scalar field, but also the non-trivial coupling with matter constrains the scalar field itself. Moreover, this opportunity gave us a chance to rule out the non-physical "inner" horizons, which differentiates our results from those previously obtained in [4],[8]. Indeed, the quadratic equation for finding the dependence $\mu = \mu(a, k, \mu_0, Qe^{a\phi_0})$ has, in general, two solutions. It is reasonable to expect that these solutions $\mu_\pm$ should be regular functions of their arguments. Moreover, because of the fact that the solution (9.3) describes the exterior region outside the matter distribution, for any values of both parameters a and k, the following condition should be satisfied: $\mu \geq \mu_0 \geq 0$. However, it turns out that while the result μ_+ always remains regular, the solution μ_-, for some physically interesting values of the parameters a and k, might either be hidden inside the mass shell $\mu_- < \mu_0$, or it might be negative or even divergent[7]. Such a behavior makes it possible to rule out the result μ_- as physically meanningless.

As we discussed in the paper, the presence of an arbitrary coupling constant a in the Lagrangian (1), provides an opportunity to study the behavior of the solutions in a different interaction regimes. Moreover, we would like to emphasize here that, in the case of the interacting fields, the use of the gauge conditions (2.3) seems to be more appropriate then the usual Schwarzchild gauge [4]-[8]. To demonstrate this, let us analyze the behavior of the horizons $h_\pm$ corresponding to the solution [8] in the limit of the strong interaction $a \to \infty$. Thus, by presenting these horizons in the equivalent form :

$$h_+ = \mu_0 + \sqrt{\mu_0 + (a^2 - 1)Q^2}, \tag{9.5a}$$

$$h_- = \frac{a^2+1}{a^2-1}\Big(\sqrt{\mu_0 + (a^2 - 1)Q^2} - \mu_0\Big), \tag{9.5b}$$

one can find that both expressions above demonstrate the same tendency when $a \to \infty$, namely: $h_+, h_- \to aQ$. However, this behavior is in conflict with the model (1) which was used to obtain the solution. Indeed, as we discussed in the paper, taking the limit $a \to \infty$ is equivalent to dropping both the Maxwell and the scalar terms from the expression (1). The remaining Lagrangian should correspond to that of general relativity with the Shwarzchild solution for a static spherically symmetric case which has one finite horizon at $h_+ = 2\mu_0$. As forour results, that, unlike to these discussed above, in the limit $a \to \infty$ from the relation (9.4a) one obtains $\mu_+|_{a\to\infty} = \mu_0$, which perfectly corresponds to the Fock solution (7.4).

Thus, for a the different values of the coupling constant a we have established the correspondence of the result obtained (9.3) to a well known solutions. It should be noted that our solution naturally contains the harmonical generalizations of those discussed in [4], [8]. Our initial goal was to find a static, spherically symmetric, harmonic solution with a regular geometrical properties, so, the analysis performed in the paper, has eliminated many of these generalizations. The most interesting result obtained in this paper is the solution for a charged dilatonic black hole (7.12) in harmonical coordinates of the Minkowski space-time. This solution represents a black hole with one horizon (and hence for the Kaluza-Klein case), on which the metric, the scalar curvature and both electromagnetic and scalar fields are regular (unlike the solution [8], which has a regular outer horizon, but those inner horizon is singular). Because of this property

[7] This analysis exludes one of the results for μ in the case of the solution with a negative physical mass $\mu_0 \to -\mu_0$ as well.

we believe that the solution (7.12) will provide an interesting framework for studying different processes in general relativity as well as in Kaluza-Klein theory.

And, finally, anticipating the possible question: whether or not the obtained solutions are stable, we would like to emphasise that the stability of the solution to Einstein-Maxwell system with an extra free scalar field Eqs.(7.1) has already been proven [14]. Also note that in [9] the stability of the solution presented in [8] was inferred for outside the outer horizon. The special studies of the charged dilatonic black hole solution Eqs.(7.12) had shown [14] that this solution is stable at least against an axial perturbations. The full analysis of this problem is currently in progress and will be reported in a subsequent publication.

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